Weyl-invariant lightlike branes and soldering of black hole space-times

E. I. Guendelman[1,*], **A. Kaganovich**[1,**], **E. Nissimov**[2,***], and **S. Pacheva**[2,#]

[1] Department of Physics, Ben-Gurion University of the Negev, P.O.Box 653, 84105 Beer-Sheva, Israel
[2] Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Boul. Tsarigradsko Chausee 72, 1784 Sofia, Bulgaria

Received 1 December 2006, accepted 5 February 2007
Published online 15 May 2007

Key words Weyl-conformal invariance, lightlike branes, black holes
PACS 11.25.-w, 04.70.-s, 04.50.+h

We consider self-consistent coupling of the recently introduced new class of Weyl-conformally invariant *lightlike* branes (*WILL*-branes) to $D = 4$ Einstein-Maxwell system plus a $D = 4$ three-index antisymmetric tensor gauge field. We find static spherically-symmetric solutions where the space-time consists of two regions with different black-hole-type geometries and different values for a *dynamically generated* cosmological constant, separated by the *WILL*-brane which "straddles" their common event horizon. Furthermore, the *WILL*-brane produces a potential "well" around itself acting as a trap for test particles falling towards the horizon.

1 Introduction

Lightlike membranes are of particular interest in general relativity as they describe impulsive lightlike signals arising in various violent astrophysical events, *e.g.*, final explosion in cataclysmic processes such as supernovae and collision of neutron stars [1]. Lightlike membranes are basic ingredients in the so called "membrane paradigm" theory [2] which appears to be a quite effective treatment of the physics of a black hole horizon.

In [3,4] lightlike membranes in the context of gravity and cosmology have been extensively studied from a phenomenological point of view, *i.e.*, by introducing them without specifying the Lagrangian dynamics from which they may originate. Recently in a series of papers [6,7] we have developed a new field-theoretic approach for a systematic description of the dynamics of lightlike branes starting from concise *Weyl-conformally invariant* actions. The latter are related to, but bear significant qualitative differences from, the standard Nambu-Goto-type p-brane actions[1] (here $(p+1)$ is the dimension of the brane world-volume).

In the present note we discuss spherically-symmetric solutions for the coupled system of bulk $D = 4$ Einstein-Maxwell plus 3-index antisymmetric tensor gauge field interacting with a *WILL*-brane. The latter serves as a matter and charged source for gravity and electromagnetism and, in addition, produces a space-varying dynamical cosmological constant. The above solutions describe space-times divided into two separate regions with different black hole geometries and different values of the dynamically generated

* E-mail: guendel@bgumail.bgu.ac.il.
** E-mail: alexk@bgumail.bgu.ac.il.
*** Corresponding author E-mail: nissimov@inrne.bas.bg, Phone: +359 2 7144 720, Fax: +359 2 975 3619
# E-mail: svetlana@inrne.bas.bg.

[1] In [5] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

cosmological constant, separated by the *WILL*-brane which automatically position itself on ("straddles") their common horizon. The matching of the physical parameters of the two black hole space-time regions ("soldering") is explicitly given in terms of the free *WILL*-brane coupling parameters (electric surface charge density and Kalb-Rammond coupling constant). A physically intersting implication of the above solutions is the emergence of a potential "well" around the *WILL*-brane trapping infalling test particles towards the common horizon.

2 Weyl-conformally invariant lightlike branes

In [6,7] we proposed the following new kind of p-brane action (in what follows we shall concentrate on the first nontrivial case $p=2$):

$$S = -\int d^3\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}}\Big] - q\int d^3\sigma\, \varepsilon^{abc}\mathcal{A}_\mu \partial_a X^\mu F_{bc} - \frac{\beta}{3!}\int d^3\sigma\, \varepsilon^{abc}\partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \mathcal{A}_{\mu\nu\lambda} \tag{1}$$

The first significant difference of (1) w.r.t. standard Nambu-Goto-type p-brane action is the presence of a new non-Riemannian reparametrization-covariant integration measure density:

$$\Phi(\varphi) \equiv \frac{1}{3!}\varepsilon_{ijk}\varepsilon^{abc}\partial_a\varphi^i\partial_b\varphi^j\partial_c\varphi^k\;,\;(a,b,c=0,1,2\;,\;i,j,k=1,2,3)\,,$$

built in terms of auxiliary world-volume scalar fields φ^i. As usual γ_{ab} denotes the intrinsic Riemannian metric on the brane world-volume and $\gamma \equiv \det\|\gamma_{ab}\|$. The second important difference is the "square-root" Maxwell term[2] involving an auxiliary world-volume gauge field A_a with $F_{ab} = \partial_a A_b - \partial_b A_a$. $G_{\mu\nu}$ ($\mu,\nu = 0,1,2,3$) denotes Riemannian metric on the embedding $D = 4$ space-time. The second Chern-Simmons-like term in (1), describing a coupling to external $D=4$ space-time electromagnetic field $\mathcal{A}_\mu$, is a special case of a class of Chern-Simmons-like couplings of extended objects to external electromagnetic fields proposed in [9]. The last term is a Kalb-Ramond-type coupling to external space-time rank 3 gauge potential $\mathcal{A}_{\mu\nu\lambda}$.

The action (1) is manifestly invariant under Weyl (conformal) symmetry: $\gamma_{ab}\longrightarrow\gamma'_{ab} = \rho\,\gamma_{ab}$, $\varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi)$ with Jacobian $\det\Big\|\frac{\partial\varphi'^{\,i}}{\partial\varphi^j}\Big\| = \rho$.

Let us recall the physical significance of $\mathcal{A}_{\mu\nu\lambda}$ [10]. In $D = 4$ when adding kinetic term for $\mathcal{A}_{\mu\nu\lambda}$ coupled to gravity (see Eq.(5) below), its field-strength $\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]} = \mathcal{F}\sqrt{-G}\varepsilon_{\kappa\lambda\mu\nu}$ with a single independent component $\mathcal{F}$ produces *dynamical (positive) cosmological constant* $K = \frac{4}{3}\pi G_N \mathcal{F}^2$.

Invariance under world-volume reparametrizations allows to introduce the standard (synchronous) gauge-fixing conditions: $\gamma^{0i} = 0$ $(i = 1,2)$, $\gamma^{00} = -1$. With the latter gauge choice and using the short-hand notation $(\partial_a X\partial_b X) \equiv \partial_a X^\mu G_{\mu\nu}\partial_b X^\nu$, the equations of motion for the brane action (1) read:

$$(\partial_0 X\partial_0 X) = 0 \quad , \quad (\partial_0 X\partial_i X) = 0 \quad , \quad \big(\partial_i X\partial_j X\big) - \frac{1}{2}\gamma_{ij}\gamma^{kl}\left(\partial_k X\partial_l X\right) = 0\;, \tag{2}$$

these are in fact constraints analogous to the (classical) Virasoro constraints of string theory;

$$\partial_i X^\mu\partial_j X^\nu \mathcal{F}_{\mu\nu}(\mathcal{A}) = 0 \quad , \quad \partial_i\chi + \sqrt{2}q\partial_0 X^\mu\partial_i X^\nu \mathcal{F}_{\mu\nu}(\mathcal{A}) = 0\;, \tag{3}$$

(here $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ plays the role of *variable brane tension*, $\mathcal{F}_{\mu\nu}(\mathcal{A}) = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$);

$$\Box^{(3)}X^\mu + \Big(-\partial_0 X^\nu\partial_0 X^\lambda + \gamma^{kl}\partial_k X^\nu\partial_l X^\lambda\Big)\Gamma^\mu_{\nu\lambda}$$

[2] "Square-root" Maxwell (Yang-Mills) action in $D=4$ was originally introduced in the first [8] and later generalized to "square-root" actions of higher-rank antisymmetric tensor gauge fields in $D \geq 4$ in the second and third [8].

$$- q \frac{\gamma^{kl} \left(\partial_k X \partial_l X\right)}{\sqrt{2}\,\chi} \partial_0 X^\nu \mathcal{F}_{\lambda\nu} G^{\lambda\mu} - \frac{\beta}{3!} \frac{\varepsilon^{abc}}{\chi\sqrt{\gamma^{(2)}}} \partial_a X^\kappa \partial_b X^\lambda \partial_c X^\nu G^{\mu\rho} \mathcal{F}_{\rho\kappa\lambda\nu} = 0 \,, \tag{4}$$

where $\mathcal{F}_{\rho\kappa\lambda\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]}$ as above, $\tilde{\Box}^{(3)} \equiv -\frac{1}{\chi\sqrt{\gamma^{(2)}}}\partial_0\left(\chi\sqrt{\gamma^{(2)}}\partial_0\right) + \frac{1}{\chi\sqrt{\gamma^{(2)}}}\partial_i\left(\chi\sqrt{\gamma^{(2)}}\gamma^{ij}\partial_j\right)$, where $\gamma^{(2)} \equiv \det\|\gamma_{ij}\|$ $(i,j = 1,2)$, and $\Gamma^\mu_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the affine connection corresponding to the external space-time metric $G_{\mu\nu}$.

The first Virasoro-like constraint in (2) explicitly exhibits the inherent lightlike property of the brane model (1), hence the acronym *WILL* (Weyl-invariant light-like) brane.

3 Bulk gravity-matter coupled to *will*-brane

Let us now consider the coupled Einstein-Maxwell-*WILL*-brane system adding also a coupling to a rank 3 gauge potential:

$$S = \int d^4x \sqrt{-G} \left[\frac{R(G)}{16\pi G_N} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4!2}\mathcal{F}_{\kappa\lambda\mu\nu}\mathcal{F}^{\kappa\lambda\mu\nu}\right] + S_{\mathrm{WILL-brane}} \,. \tag{5}$$

Here $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$, $\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]} = \mathcal{F}\sqrt{-G}\varepsilon_{\kappa\lambda\mu\nu}$ as above, and the *WILL*-brane action is the same as in (1).

The equations of motion for the *WILL*-brane subsystem are the same as (2)–(4), whereas the equations for the space-time fields read:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi G_N \left(T^{(EM)}_{\mu\nu} + T^{(\mathrm{rank-3})}_{\mu\nu} + T^{(\mathrm{brane})}_{\mu\nu}\right) \,, \tag{6}$$

$$\partial_\nu\left(\sqrt{-G}G^{\mu\kappa}G^{\nu\lambda}\mathcal{F}_{\kappa\lambda}\right) + q\int d^3\sigma\,\delta^{(4)}\Big(x - X(\sigma)\Big)\varepsilon^{abc}F_{bc}\partial_a X^\mu = 0 \,, \tag{7}$$

$$\varepsilon^{\lambda\mu\nu\kappa}\partial_\kappa\mathcal{F} + \beta\int d^3\sigma\,\delta^{(4)}(x - X(\sigma))\varepsilon^{abc}\partial_a X^\lambda\partial_a X^\mu\partial_a X^\nu = 0 \,. \tag{8}$$

The energy-momentum tensors read: $T^{(EM)}_{\mu\nu} = \mathcal{F}_{\mu\kappa}\mathcal{F}_{\nu\lambda}G^{\kappa\lambda} - G_{\mu\nu}\frac{1}{4}\mathcal{F}_{\rho\kappa}\mathcal{F}_{\sigma\lambda}G^{\rho\sigma}G^{\kappa\lambda}$,

$$T^{(\mathrm{rank-3})}_{\mu\nu} = \frac{1}{3!}\left[\mathcal{F}_{\mu\kappa\lambda\rho}\mathcal{F}_\nu{}^{\kappa\lambda\rho} - \frac{1}{8}G_{\mu\nu}\mathcal{F}_{\kappa\lambda\rho\sigma}\mathcal{F}^{\kappa\lambda\rho\sigma}\right] = -\frac{1}{2}\mathcal{F}^2 G_{\mu\nu} \,, \tag{9}$$

$$T^{(\mathrm{brane})}_{\mu\nu} = -G_{\mu\kappa}G_{\nu\lambda}\int d^3\sigma\,\frac{\delta^{(4)}\Big(x - X(\sigma)\Big)}{\sqrt{-G}}\,\chi\sqrt{-\gamma}\gamma^{ab}\partial_a X^\kappa\partial_b X^\lambda \,. \tag{10}$$

For the bulk gravity-matter system coupled to a charged *WILL*-brane (5) we find the following static spherically symmetric solutions. The bulk space-time consists of two regions separated by the *WILL*-brane sitting on ("straddling") a common horizon of the former:

$$(ds)^2 = -A_{(\mp)}(r)(dt)^2 + \frac{1}{A_{(\mp)}(r)}(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2] \,, \tag{11}$$

where the subscript $(-)$ refers to the region inside, whereas the subscript $(+)$ refers to the region outside the horizon at $r = r_0 \equiv r_{\mathrm{horizon}}$ with $A_{(\mp)}(r_0) = 0$. The interior region is a Schwarzschild-de-Sitter space-time:

$$A(r) \equiv A_{(-)}(r) = 1 - K_{(-)}r^2 - \frac{2G_N M_{(-)}}{r} \,, \quad \text{for } r < r_0 \,, \tag{12}$$

whereas the exterior region is Reissner-Norström-de-Sitter space-time:

$$A(r) \equiv A_{(+)}(r) = 1 - K_{(+)}r^2 - \frac{2G_N M_{(+)}}{r} + \frac{G_N Q^2}{r^2} \quad , \quad \text{for} \;\; r > r_0 \; , \tag{13}$$

with Reissner-Norström (squared) charge given by $Q^2 = 8\pi q^2 r_0^4$. The rank 3 tensor gauge potential together with its Kalb-Rammond-type coupling to the *WILL*-brane produce via Eq.(8) a dynamical space-varying cosmological constant which is different inside and outside the horizon: $K_{(\pm)} = \frac{4}{3}\pi G_N \mathcal{F}^2_{(\pm)}$ for $r \geq r_0 \;\; (r \leq r_0)$, $\mathcal{F}_{(+)} = \mathcal{F}_{(-)} - \beta$. The Einstein Eqs.(6) and the X^μ-brane Eqs.(4) yield two matching conditions for the normal derivatives w.r.t. the horizon of the space-time metric components:

$$\left(\partial_r A_{(+)} - \partial_r A_{(-)}\right)\Big|_{r=r_0} = -16\pi G_N \chi \; , \; \left(\partial_r A_{(+)} - \partial_r A_{(-)}\right)\Big|_{r=r_0} = -\frac{r_0(2q^2+\beta^2)\partial_r A_{(-)}\,\big|_{r=r_0}}{2\chi + \beta r_0 \mathcal{F}_{(-)}} \; .$$

The latter conditions allow to express all physical parameters of the solution, *i.e.*, two spherically symmetric black hole space-time regions "soldered" along a common horizon via the *WILL*-brane in terms of 3 free parameters $(q, \beta, \mathcal{F})$ where (cf. Eq.(1)): (a) q is the *WILL*-brane surface electric charge density; (b) β is the *WILL*-brane (Kalb-Rammond-type) charge w.r.t. rank 3 space-time gauge potential $\mathcal{A}_{\lambda\mu\nu}$; (c) $\mathcal{F}_{(-)}$ is the vacuum expectation value of the 4-index field-strength $\mathcal{F}_{\kappa\lambda\mu\nu}$ in the interior region. For the common horizon radius, the Schwarzschild and Reissner-Nordström masses we obtain:

$$r_0^2 = \frac{1}{4\pi G_N \left(\mathcal{F}^2_{(-)} - \beta\mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2}\right)} \; ,$$

$$M_{(-)} = \frac{r_0\left(\frac{2}{3}\mathcal{F}^2_{(-)} - \beta\mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2}\right)}{2G_N\left(\mathcal{F}^2_{(-)} - \beta\mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2}\right)} \; , \tag{14}$$

$$M_{(+)} = M_{(-)} + \frac{r_0}{2G_N\left(\mathcal{F}^2_{(-)} - \beta\mathcal{F}_{(-)} + q^2 + \frac{\beta^2}{2}\right)}\left(2q^2 + \frac{2}{3}\beta\mathcal{F}_{(-)} - \frac{1}{3}\beta^2\right) \; . \tag{15}$$

For the brane tension we get accordingly: $\chi = \frac{r_0}{2}\left(q^2 + \frac{\beta^2}{2} - 2\beta\mathcal{F}_{(-)}\right)$.

Using expressions (14), (15) we find for the slopes of the metric coefficients $A_{(\pm)}(r)$ at $r = r_0$:

$$\partial_r A_{(+)}\big|_{r=r_0} = -\partial_r A_{(-)}\big|_{r=r_0} \; , \; \partial_r A_{(-)}\big|_{r=r_0} = 8\pi G_N \chi = 4\pi G_N r_0\left(q^2 + \frac{\beta^2}{2} - 2\beta\mathcal{F}_{(-)}\right) \; . \tag{16}$$

In view of (16) (and assuming for definiteness $\beta > 0$) we conclude:

(i) In the area of parameter space $\mathcal{F}_{(-)} > \frac{q^2+\frac{\beta^2}{2}}{2\beta}$ (*i.e.*, when $\chi < 0$ – negative brane tension) the common horizon is: (a) the de-Sitter horizon from the point of view of the interior Schwarzschild-de-Sitter geometry; (b) it is the external Reissner-Nordström horizon (the larger one) from the point of view of the exterior Reissner-Nordström-de-Sitter geometry.

(ii) In the opposite area of parameter space $\mathcal{F}_{(-)} < \frac{q^2+\frac{\beta^2}{2}}{2\beta}$ (*i.e.*, when $\chi > 0$ – positive brane tension) the common horizon is: (a) the Schwarzschild horizon from the point of view of the interior Schwarzschild-de-Sitter geometry; (b) it is the internal (the smaller one) Reissner-Nordström horizon from the point of view of the exterior Reissner-Nordström-de-Sitter geometry.

Now let us consider planar motion of a (charged) test patricle with mass m and electric charge q_0 in a gravitational background given by the solutions in Sect. 3. Conservation of energy yields $\frac{E^2}{m^2} =$

${r'}^2 + V^2_{eff}(r)$ (E, J – energy and orbital momentum of the test particle; prime indicates proper-time derivative) with:

$$V^2_{eff}(r) = A_{(-)}(r)\left(1+\frac{J^2}{m^2r^2}\right) + \frac{2Eq_0}{m^2}\sqrt{2}qr_0 - \frac{q_0^2}{m^2}2q^2r_0^2 \qquad (r \le r_0)$$

$$V^2_{eff}(r) = A_{(+)}(r)\left(1+\frac{J^2}{m^2r^2}\right) + \frac{2Eq_0}{m^2}\frac{\sqrt{2}qr_0^2}{r} - \frac{q_0^2}{m^2}\frac{2q^2r_0^4}{r^2} \qquad (r \ge r_0) \tag{17}$$

where $A_{(\mp)}$ are the same as in (12) and (13). Taking into account (16) we see that in the parameter interval $\mathcal{F}_{(-)} \in \left(\frac{q^2+\frac{\beta^2}{2}}{\beta}, \infty\right)$ the (squared) effective potential $V^2_{eff}(r)$ acquires a potential "well" in the vicinity of the *WILL*-brane (the common horizon) with a minimum on the brane itself.

In the simplest physically interesting case with $q = 0$, $\mathcal{F}_{(-)} = \beta$ and β – arbitrary, *i.e.*, matching of Schwarzschild-de-Sitter interior (with dynamically generated cosmological constant) against pure Schwarzschild exterior (with *no* cosmological constant) along the *WILL*-brane as their common horizon, the typical form of $V^2_{eff}(r)$ is graphically depicted in Fig. 1.

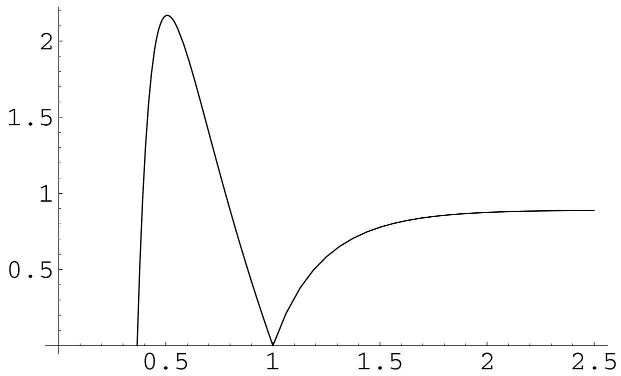

Fig. 1 Shape of $V^2_{eff}(r)$ as a function of the dimensionless ratio $x \equiv r/r_0$.

Thus, we conclude that if a test particle moving towards the common event horizon loses energy (*e.g.*, by radiation), it may fall and be trapped by the potential well, so that it neither falls into the black hole nor can escape back to infinity and, as a result, a "cloud" of trapped particles is formed around the *WILL*-brane materialized horizon.

Acknowledgements E.N. and S.P. are supported by European RTN network *"Constituents, Fundamental Forces and Symmetries of the Universe"* (contract No.*MRTN-CT-2004-005104*). They also received partial support from Bulgarian NSF grant *F-1412/04*. Finally, all of us acknowledge support of our collaboration through the exchange agreement between the Ben-Gurion University of the Negev (Beer-Sheva, Israel) and the Bulgarian Academy of Sciences.

References

[1] C. Barrabés and P. Hogan, Singular Null-Hypersurfaces in General Relativity (World Scientific, Singapore, 2004).
[2] K. Thorne, R. Price, and D. Macdonald (eds.), Black Holes: The Membrane Paradigm (Yale University Press, New Haven, CT, 1986).
[3] W. Israel, Nuovo Cim. B **44**, 1 (1966); erratum, ibid **48**, 463 (1967).
[4] C. Barrabés and W. Israel, Phys. Rev. D **43**, 1129 (1991); T. Dray and G. 't Hooft, Class. Quantum Gravity **3**, 825 (1986).
[5] C. Barrabés and W. Israel, Phys. Rev. D **71**, 064008 (2005) (gr-qc/0502108).

[6] E. Guendelman, A. Kaganovich, E. Nissimov, and S. Pacheva, hep-th/0409078; in: Second Workshop on Gravity, Astrophysics and Strings edited by P. Fiziev et.al. (Sofia Univ. Press, Sofia, Bulgaria, 2005) (hep-th/0409208); in: Third Internat. School on Modern Math. Physics, Zlatibor (Serbia and Montenegro), edited by B. Dragovich and B. Sazdovich (Belgrade Inst. Phys. Press, 2005) (hep-th/0501220).

[7] E. Guendelman, A. Kaganovich, E. Nissimov, and S. Pacheva, Phys. Rev. D **72**, 0806011 (2005) (hep-th/0507193); hep-th/0611022.

[8] H. B. Nielsen and P. Olesen, Nucl. Phys. B **57**, 367 (1973); A. Aurilia, A. Smailagic, and E. Spallucci, Phys. Rev. D **47**, 2536 (1993) (hep-th/9301019); A. Aurilia and E. Spallucci, Class. Quantum Gravity **10** 1217 (1993).

[9] A. Davidson and E. Guendelman, Phys. Lett. B **251**, 250 (1990).

[10] A. Aurilia, H. Nicolai, and P. Townsend, Nucl. Phys. B **176**, 509 (1980); A. Aurilia, Y. Takahashi, and P. Townsend, Phys. Lett. B **95**, 265 (1980).

